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The Bispectrum and Higher-Order Spectra: A Bibliography

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The Bispectrum and Higher-Order Spectra: A Bibliography

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The bispectrum or Fourier transform of the 3rd order moments of a time series is useful for the study of nonlinear or non-Gaussian phenomena. This bibliography cites 134 papers covering both theory and application. The entries are classified by content with special effort made to indicate papers that contain material on the computation, display and interpretation of the bispectrum.

Key Words: Bispectrum; cumulant spectra; nonlinear time series; polyspectra; spectrum analysis; statistics; time series.

INTRODUCTION

The bispectrum or Fourier transform of 3rd order moments is useful for the study of nonlinear structure in time series. It is sensitive to phase coherence between wave components due, for example, to nonlinear wave interactions or to the harmonic structure of periodic functions that are not pure sinusoids. The bispectrum is useful both for detecting the presence of phase coherent structure and measuring the fraction of spectral power due to coherent components.

The bispectrum was first discussed by Tukey (1953). The general idea of harmonic analysis of higher order moments was also introduced by Blanc-Lapierre and Fortet (1953). Magness (1954) used what would now be called the trispectrum (transform of 4th order moments) to compute the spectral response of a quadratic device to non-Gaussian noise. Mazelsky (1954) used higher order spectra to determine non-Gaussian probability functions for the input disturbances and output responses of linear systems. Tick (1961) showed how the cross-bispectrum could be used to estimate the transfer function of a quadratic system. Following the suggestion of Tukey (1963) and Tick (1963), bispectral analysis was first applied to the study of nonlinear phenomena in ocean waves by Hasselmann, Munk and MacDonald (1963). The first rather thorough discussion of the bispectrum and its properties appears in Shaman (1964) which extends Tukey's 1953 unpublished manuscript. Between 1965 and 1967 the general theory of the bispectrum and higher order cumulant spectra (or polyspectra) was intensively developed by Brillinger, Rosenblatt, Van Ness, Godfrey, Parzen, and Akaike. Since 1967 the bispectrum has found a wide variety of applications primarily in the physical sciences. However, the practice of bispectral analysis has received little attention in the statistical literature in more than a decade.

The purpose of this bibliography is to make the literature of the last two decades, now widely scattered in physical sciences journals, more accessible. There may well be other references to harmonic analysis of higher order moments in the literature of stochastic processes. In particular, the vast literature on kernel estimation and identification methods has not been searched for related papers. However, four such papers that are of special interest are included. Hung and Stark (1977) is a review of kernel identification methods with an extensive bibliography (88 entries). Yasui (1979) discusses the application of kernel methods to nonlinear systems analysis, and also has an extensive bibliography (49 entries). Both papers mention the relationship to the bispectrum. Hung and Stark (1979), and Hung, Brillinger, and Stark (1979) use bispectral analysis to compute higher order kernels for nonlinear systems.

The bibliography is arranged alphabetically by first author. There is a chronological listing of first authors and an alphabetical cross reference of second authors with first authors. Finally, the entries are classified by content. Although this is not a review paper, an effort has been made to indicate papers that contain material on the computation, display and interpretation of the bispectrum. Huber, Kleiner, Gasser, and Dummermuth (1971), and Kim and Powers (1979) give nice reviews of the bispectrum and its interpretation.

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Bibliography

- Akaike, H. (1966). Note on higher order spectra. *Annals of the Institute of Statistical Mathematics*, 18, 123-126.
- Akaike, H., Arahata, E., and Ozaki, T. (1975). TIMSAC-74, A time series analysis and control program package (1). *Computer Science Monograph*, No. 5, The Institute of Statistical Mathematics, Tokyo, Japan.
- Akaike, H., Arahata, E., and Ozaki, T. (1976). TIMSAC-74, A time series analysis and control program package (2). *Computer Science Monograph*, No. 6, The Institute of Statistical Mathematics, Tokyo, Japan.
- Alekseyev, V. G. (1970). Accuracy of experimental determination of higher-order moments of time series. *Engineering Cybernetics*, 6, 1195-1201.
- Armstrong, J. W. (1977). Bispectral analysis of meter wavelength interplanetary scintillation. *Astronomy and Astrophysics*, 61, 313-320.
- Aubry, M. P. (1966). Application de l'analyse bispectrale a l'etude de la diffraction. *Annales d'Astrophysique*, 29, 389-406 (In French).
- Aubry, M. P. (1967). Application de l'analyse bispectrale a l'etude de la diffraction. *Annales d'Astrophysique*, 30, 101-110 (In French).
- Barnett, T., Johnson, L. C., Naitoh, P., Hicks, N., and Nute, C. (1971). Bispectrum analysis of electroencephalogram signals during waking and sleeping. *Science*, 172, 401-402.
- Bartlett, M. S. (1967). Some remarks on the analysis of time series. *Biometrika*, 54, 25-38.
- Blanc-Lapierre, A., and Fortet R. (1953). *Theory of Random Functions*, Vol II. Gordon and Breach, New York (1965 Translation of 1953 French Edition).
- Borresen, R. (1978). Experimental determination of the quadratic transfer function governing slowly oscillating phenomena in irregular waves. *Proceedings Offshore Technology Conference*, 10, 457-464.
- Bozzi Zadro, M., and Caputo, M. (1968). Spectral, bispectral analysis and Q of the free oscillations of the earth. *Supplemento al Nuovo Cimento*, 6, 67-81.
- Brillinger, D. R. (1965). An introduction to polyspectra. *Annals of Mathematical Statistics*, 36, 1351-1374.
- Brillinger, D. R., and Rosenblatt, M. (1967a). Asymptotic Theory of Estimates of k-Th Order Spectra. *Spectral Analysis of Time Series*, (B. Harris, Ed.), Wiley, New York, 153-188.

- Brillinger, D. R., and Rosenblatt, M. (1967b). Computation and Interpretation of k-th Order Spectra. Spectral Analysis of Time Series, (B. Harris, Ed.), Wiley, New York, 189-232.
- Brillinger, D. R. (1970). The identification of polynomial systems by means of higher order spectra. Journal of Sound and Vibration, 12, 301-313.
- Brillinger, D. R. (1972). The spectral analysis of stationary interval functions. Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability. 1, (L. LeCam, Ed.), University of California Press, Berkeley, 483-513.
- Brillinger, D. R. (1973). An empirical investigation of the Chandler wobble and two proposed excitation processes. Bulletin of the International Statistical Institute, 45, 413-434.
- Brillinger, D. R. (1974a). Cross-spectral analysis of processes with stationary increments including the stationary G/G/ ∞ queue. Annals of Probability, 2, 815-827.
- Brillinger, D. R. (1974b). Fourier analysis of stationary processes. Proceedings of the IEEE, 62, 1628-1643.
- Brillinger, D. R. (1975). Time Series: Data Analysis and Theory. Holt, Rinehart and Winston, Inc., New York, N. Y.
- Brillinger, D. R. (1977). The identification of a particular nonlinear time series system. Biometrika, 64, 509-515.
- Briscoe, M. G. (1976). Bispectra of oceanic internal waves. Bulletin of the American Meteorological Society, 57, 113 (Abstract).
- Cartwright, D. E. (1968). A unified analysis of tides and surges round north and east Britain. Philosophical Transactions, Royal Society of London, Series A, 263, 1-55.
- Dalzell, J. F. (1972a). Application of cross-bispectral analysis to ship resistance in waves. Stevens Institute of Technology, Davidson Laboratory, Hoboken, New Jersey, Rpt. No. SIT-DL-72-1606.
- Dalzell, J. F. (1972b). Some additional studies of the application of cross-bispectral analysis to ship resistance in waves. Stevens Institute of Technology, Davidson Laboratory, Hoboken, New Jersey, Rpt. No. SIT-DL-72-1641.
- Davies, R. B. (1977). Testing the hypothesis that a point process is Poisson. Advances in Applied Probability, 9, 724-746.
- Dubkov, A. A., and Malakhov, A. N. (1978). Statistics of generalized telegraph signals. Radiophysics and Quantum Electronics, 21, 54-58.

- Dumermuth, G., Huber, P. J., Kleiner, B., and Gasser, T. (1970). Numerical analysis of electroencephalographic data. *IEEE Transactions on Audio and Electroacoustics*, AU-18, 404-411.
- Dumermuth, G., Huber, P. J., Kleiner, B., and Gasser, T. (1971). Analysis of the interrelations between frequency bands of the EEG by means of the bispectrum. A preliminary study. *Electroencephalography and Clinical Neurophysiology*, 31, 137-148.
- Dumermuth, G., and Gasser, T. (1978). Computation of EEG bi-spectra. *Computer Programs in Biomedicine*, 8, 235-242.
- Feuerverger, A. (1972). On the cumulant spectra approach to polynomial regression of stationary time series. Doctoral dissertation, University of California, Berkeley.
- Fried, D. L. (1979). Angular dependence of the atmospheric turbulence effect in speckle interferometry. *Optica Acta* 26, 597-613.
- Gabrielli, C., Keddam, M., and Raillon, L. (1979). Random signals: third-order correlation-measurement. *Journal of Physics, Section E, Scientific Instruments*, 12, 632-636.
- Gasser, T. (1972). System identification, polyspectra and related functions. Doctoral dissertation, Swiss Federal Institute of Technology.
- Gasser, T. (1975). Goodness-of-fit tests for correlated data. *Biometrika*, 62, 563-570.
- Gerzon, M. A. (1978). Mathematics and sound perception. *Journal of the Audio Engineering Society*, 26, 46-50.
- Godfrey, M. D. (1965). An exploratory study of the bispectrum of economic time series. *Journal of the Royal Statistical Society, Series C, Applied Statistics*, 14, 48-69.
- Hasselmann, K., Munk, W., and MacDonald, G. (1963). Bispectra of Ocean Waves. *Time Series Analysis*, (M. Rosenblatt, Ed.), Wiley, New York, 125-139.
- Hasselmann, K. (1966). On nonlinear ship motions in irregular waves. *Journal of Ship Research*, 10, 64-68.
- Hasselmann, D. E. (1978). Wind-wave generation by energy and momentum flux to the forced components of a wave field. *Journal of Fluid Mechanics*, 85, 543-572.
- Haubrich, R. A., (1965). Earth noise, 5 to 500 millicycles per second. *Journal of Geophysical Research*, 70, 1415-1427.

Helland, K. N., and Van Atta, C. W. (1976). Response of constant-current and constant-temperature anemometers to artificial turbulence. *Physics of Fluids*, 19, 1109-1117.

Helland, K. N., Lii, K. S., and Rosenblatt, M. (1977). Bispectra of atmospheric and wind tunnel turbulence. *Applications of Statistics*, 223-248.

Helland, K. N., Lii, K. S., and Rosenblatt, M. (1979). Bispectra and energy transfer in grid-generated turbulence. *Developments in Statistics*, Vol. 2, Academic Press, New York, 123-155.

Herring, J. R. (1980). Theoretical calculations of turbulent bispectra. *Journal of Fluid Mechanics*, 97, 193-204.

Hinich, M. J., and Clay, C. S. (1968). The application of the discrete Fourier transform in the estimation of power spectra, coherence, and bispectra of geophysical data. *Reviews of Geophysics*, 6, 347-363.

Hinich, M. J. (1979). Estimating the lag structure of a nonlinear time series model. *Journal of the American Statistical Association*, 74, 449-452.

Huber, P. J., Kleiner, B., Gasser, T., and Dumermuth, G. (1971). Statistical methods for investigating phase relations in stationary stochastic processes. *IEEE Transactions Audio & Electroacoustics*, AU-19, 78-86.

Hung, G., and Stark, L. (1977). The kernel identification method (1910-1977)- review of theory, calculation, application, and interpretation. *Mathematical Biosciences*, 37, 135-190.

Hung, G., Brillinger, D. R., and Stark, L. (1979). Interpretation of kernels II. Same-Signed 1st-and 2nd-degree (main-diagonal) kernels of the human pupillary system. *Mathematical Biosciences*, 46, 159-187.

Hung, G., and Stark, L. (1979). Interpretation of kernels. III. Positive off-diagonal kernels as correlates of the dynamic process of pupillary escape. *Mathematical Biosciences*, 46, 189-203.

Kedem-Kimelfeld, B. (1975). Estimating the lags of lag processes. *Journal of the American Statistical Association*, 70, 603-605.

Kim, Y. C., and Powers, E. J. (1976). Bispectral wave analysis of nonlinear wave-wave interactions in plasmas. *IEEE International Conference on Plasma Science (Abstracts)*, 78.

Kim, Y. C., and Powers, E. J. (1977). Experimental determination of harmonic generation coupling coefficients using bispectral analysis. *IEEE International Conference on Plasma Science (Abstracts)*, 154.

- Kim, Y. C., and Powers, E. J. (1978). Digital bispectral analysis of self-excited fluctuation spectra. *Physics of Fluids*, 21, 1452-1453.
- Kim, Y. C., and Powers, E. J. (1979). Digital bispectral analysis and its applications to nonlinear wave interactions. *IEEE Transactions on Plasma Science*, PS-7, 120-131.
- Kim, Y. C., Beall, J. M., Powers, E. J., and Miksad, R. W. (1980). Bispectrum and nonlinear wave coupling. *Physics of Fluids*, 23, 258-263.
- Kiriyama, K., and Sato, T. (1972). On a bispectrum synthesizer. *Bulletin Tokyo Institute of Technology*, No. 112, 9-25.
- Kleiner, B. (1971). Die berechnung von bispecktren. Doctoral dissertation, Eidgenossischen Technischen Hochschule, Zurich.
- Korein, J., Tick, L. J., Zeitlin, R. A., and Randt, C. T. (1968). Linear and nonlinear spectral analytic techniques applied to the human electroencephalogram. *New York Academy of Science, Bulletin*, 44, 1126-1128.
- Leonov, V. P. (1964). Some Applications of Higher-order Semi-invariants to the Theory of Stationary Random Processes. *Izdatilstvo, Nauka* (In Russian).
- Lii, K. S., Rosenblatt, M., and Van Atta, C. W. (1976). Bispectral measurements in turbulence. *Journal of Fluid Mechanics*, 77, 45-62.
- Lumley, J. L., and Takeuchi, K. (1976). Application of central-limit theorems to turbulence and higher-order spectra. *Journal of Fluid Mechanics*, 74, 433-468.
- MacDonald, G. J. F. (1963). The bispectra of atmospheric pressure records. *Proceedings IBM Scientific Computing Symposium on Statistics*. IBM, White Plains, New York, 247-264.
- Madden, T. (1964). Spectral, cross-spectral, and bispectral analysis of low frequency electromagnetic data. *Natural Electromagnetic Phenomena Below 30 KC/S*. Plenum Press, New York, 429-450.
- Mager, P. P. (1974). Employment of time series theory in experimental medicine. *Acta Histochemica*, 49, 233-245 (In German).
- Mager, P. P. (1975). The discrimination in time series analysis - A working procedure. *Activitas nervosa superior*, 17, 149-154.
- Magness, T. A. (1954). Spectral response of a quadratic device to non-Gaussian noise. *Journal of Applied Physics*, 25, 1357-1365.

Marussi, A., Bozzi Zadro, M., and Manzoni, G. (1968). Nonlinear elasticity in the free oscillations of the earth as revealed by spectral and bispectral analysis. Trieste University, Institute of Geodesy and Geophysics, Report No. AD-685042.

Mazelsky, B. (1954). Extension of power spectral methods of generalized harmonic analysis to determine non-Gaussian probability functions of random input disturbances and output responses of linear systems. Journal of the Aeronautical Sciences, 21, 145-153.

McComas, C. H. III (1978). Bispectra of internal waves. Technical Report, WHOI-78-25, Woods Hole Oceanographic Institution.

McComas, C. H. and Briscoe, M. G. (1980). Bispectra of internal waves. Journal of Fluid Mechanics, 97, 205-213.

Mitsuishi, A., and Nakashima, S. N. (1972). Raman scattering in solids. Butsuri (Japan), 27, 815-830.

Murata, T., and Ohara, H. (1977). Identification of two unknown signals. Transactions Institute of Electronics and Communication Engineers of Japan, Sect. E, E60 (Abstract).

Murty, T. S., and Henry, R. F. (1972). Some tsunami studies for the west coast of Canada. Manuscript Rep. Ser. 28, Can. Mar. Sci. Dir., Ottawa, Ont.

Neshyba, S., and Sobey, E. J. C. (1975). Vertical cross coherence and cross bispectra between internal waves measured in a multiple-layered ocean. Journal of Geophysical Research, 80, 1152-1162.

Ohta, M., Hatakeyama, K., Hiromitsu, S., and Yamaguchi, S. (1975). A unified study on the output probability distribution of arbitrary linear vibratory systems with arbitrary random excitation. Journal of Sound and Vibration, 43, 693-711.

Ohta, M., Yamaguchi, S., and Iwashige, H. (1977). A statistical theory for road traffic noise based on the composition of component response waves and its experimental confirmation. Journal of Sound and Vibration, 52, 587-601.

Ohta, M., Yamaguchi, S., and Hiromitsu, S. (1978). A unified expression for the multivariate joint probability density function of the output fluctuation of an arbitrary linear vibratory system with arbitrary random excitation. Journal of Sound and Vibration, 56, 229-241.

Parzen E. (1967). Time Series Analysis for Models of Signal Plus White Noise. Spectral Analysis of Time Series, (B. Harris, Ed.), Wiley, New York, 233-257.

Powers, E. J., and Kim, Y. C. (1977). Determination of nonlinear wave-wave interaction coupling coefficients using bispectral analysis techniques. Bulletin of the American Physical Society, 22, 1102 (Abstract).

- Rao, S. T., Czapski, U., and Sedefian, L. (1977). Characteristics of internal oscillations in Lake Ontario. *Journal of Geophysical Research*, 82, 1725-1734.
- Roden, G. I., and Bendiner, D. J. (1973). Bispectra and cross-bispectra of temperature, salinity, sound velocity and density fluctuations with depth off northeastern Japan. *Journal of Physical Oceanography*, 3, 308-317.
- Rosenblatt, M. (1964). Some nonlinear problems arising in the study of random processes. *Radio Science Journal of Research NBS/USNC-URSI*, 68D, 933-936.
- Rosenblatt, M., and Van Ness, J. W. (1965). Estimation of the bispectrum. *Annals of Mathematical Statistics*, 36, 1120-1136.
- Rosenblatt, M. (1966). Remarks on higher order spectra. *Multivariate Analysis*, (P. R. Krishnaiah, Ed.) Academic Press, New York, 383-389.
- Rosenblatt, M. (1971). Curve estimates. *The Annals of Mathematical Statistics*, 42, 1815-1842.
- Rosenblatt, M. (1978). Energy transfer for the Burgers' equation. *Physics of Fluids*, 21, 1694-1697.
- Rosenblatt, M. (1980). Linear processes and bispectra. *Journal of Applied Probability*, 17, 265-270.
- Sasaki, K., Sato, T., and Adachi, T. (1972). Bispectrum synthesizer using multiple Poisson processes. *Bulletin Tokyo Institute of Technology*, No. 113, 55-66.
- Sasaki, K., Sato, T., and Yamashita, Y. (1975). Minimum bias windows for bispectral estimation. *Journal of Sound and Vibration*, 40, 139-148.
- Sasaki, K., Sato, T., and Nakamura, Y. (1977). Holographic passive sonar. *IEEE Transactions on Sonics and Ultrasonics*, SU-24, 193-200.
- Sasaki, K., Sato, T., and Nakamura, Y. (1978). An effective utilization of spectral spread in holographic passive imaging systems. *IEEE Transactions on Sonics & Ultrasonics*, SU-25, 177-184.
- Sasaki, K., and Sato, T. (1979). A bispectral synthesizer. *Journal of the Acoustical Society of America*, 65, 732-739.
- Sasaki, O., Sato, T., and Oda, T. (1980). Laser doppler vibration measuring system using bispectral analysis. *Applied Optics*, 19, 151-153.
- Sato, T., Sasaki, K., and Mori, A. (1975). Statistical properties of wind over the coast and its stochastic model. *Journal of the Acoustical Society of America*, 57, 976-978.

- Sato, T., and Sasaki, K. (1977). Machine diagnosis by using bispectral analysis of noises. *Journal of Japan Society of Lubrication Engineers*, 22, 632-636 (In Japanese).
- Sato, T., Sasaki, K., and Nakamura, Y. (1977). Real-time bispectral analysis of gear noise and its application to contactless diagnosis. *Journal of the Acoustical Society of America*, 62, 382-387.
- Sato, T., and Sasaki, K. (1977). Bispectral holography. *Journal of the Acoustical Society of America*, 62, 404-408.
- Sato, T., Kishimoto, T., and Sasaki, K. (1978). Laser doppler particle measuring system using nonsinusoidal forced vibration and bispectral analysis. *Applied Optics*, 17, 667-670.
- Sato, T., and Sasaki, O. (1978). New 3-D laser doppler velocimeter using cross-bispectral analysis. *Applied Optics*, 17, 3890-3894.
- Sato, T., Sasaki, K., and Nonaka, M. (1978). Prototype of bispectral passive imaging systems aiming machine-system diagnosis. *Journal of the Acoustical Society of America*, 63, 1611-1616.
- Sato, T., and Sasaki, O. (1979). Ultrasonic doppler velocimeter using cross-bispectral analysis. *Ultrasonic Imaging*, 1, 144-153.
- Sclove, S. L. (1978). Testing independence of variates in an infinitely divisible random vector. *Journal of Multivariate Analysis*, 8, 479-485.
- Shaman, P. (1964). Bispectral analysis of stationary time series. Scientific Paper No. 18, New York University, School of Engineering and Science, Statistical Laboratory.
- Shimizu, H., and Inoue, T. (1978). Machine fault diagnosis by vibrational analysis. Exploratory introduction of bispectrum method. *Bulletin of the Faculty of Engineering, Yokohama National University*, 27, 51-60.
- Shiryaev, A. N. (1960). Some problems in the spectral theory of higher order moments. I. *Theory of Probability and its Applications*, 5, 265-284.
- Sinai, Ya. G. (1963). On higher order spectral measures of ergodic stationary processes. *Theory of Probability and its Applications*, 8, 429-436.
- Tachi, S. (1973). Separation of a periodic signal in noise by bispectrum analysis. *Transactions of the Society of Instrument and Control Engineers (Japan)*, 9, 729-738 (In Japanese).
- Tachi, S. (1975). An estimation method of a linear dynamic system by means of bispectrum analysis. *Transactions of the Society of Instrument and Control Engineers (Japan)*, 11, 729-734 (In Japanese).

Tanaka, K., Kikkawa, S., and H. Ohara. (1978). Geometrical consideration on the problem of identifying two unknown signals. Transactions of the Institute of Electronics and Communication Engineers of Japan, Sect. E, E61, No. 12, 981 (Abstract).

Ten Hoopen, M., and Zandt, P. A. (1976). Le Bi-spectre: Quelques proprietes en rapport avec des rythmes multiples. *Acustica*, 35, 303-309 (In French).

Ten Hoopen, M., and Zandt, P. A. (1977). Second-order correlation functions and bi-spectra in biological-rhythm research. *Mathematical Biosciences*, 33, 193-212.

Tick, L. J. (1961). The estimation of transfer functions of quadratic systems. *Technometrics*, 3, 563-567.

Tick, L. J. (1963). Nonlinear Probability Models of Ocean Waves. *Ocean Wave Spectra*. National Academy of Sciences, Prentice-Hall, Englewood Cliffs, New Jersey.

Tukey, J. W. (1953). The spectral representation and transformation properties of the higher moments of stationary time series. Unpublished Manuscript.

Tukey, J. W. (1959). An Introduction to the Measurement of Spectra. *Probability & Statistics* (U. Grenander Ed.), Wiley, New York, 300-330.

Tukey, J. W. (1963). What Can Data Analysis and Statistics Offer Today. *Ocean Wave Spectra*. National Academy of Sciences, Prentice-Hall, Englewood Cliffs, New Jersey.

Ueno, T., Tachi, S., Yamada, I., and Fujimura, S. (1976). Measurement of bispectrum and its applications. *Oyo Buturi* (Japan), 45, 384-396 (In Japanese).

Ueno, T., and Nakajima, T. (1977). Bispectrum analysis of surface roughness wave-forms. *Proceedings 3rd International Conference on Production Engineering*, 255-260.

Van Atta, C. W., and Yeh, T. T. (1970). Some measurements of multi-point time correlations in grid turbulence. *Journal of Fluid Mechanics*, 41, Part 1, 169-178.

Van Atta, C. W. (1974). Sampling techniques in turbulence measurements. *Annual Review of Fluid Mechanics*, 6, 75-91.

Van Atta, C. W. (1979). Inertial range bispectra in turbulence. *Physics of Fluids*, 22, 1440-1442.

Van Ness, J. W. (1966a). Asymptotic normality of bispectral estimates. *Annals of Mathematical Statistics*, 37, 1257-1272.

- Van Ness, J. W. (1966b). Empirical nonlinear prediction and polyspectra. Technical Report No. 18, Dept. of Statistics, Stanford University.
- Westcott, M. (1970). Identifiability in linear processes. *Zeitschrift fuer Wahrscheinlichkeitstheorie und Verwandte Gebiete*, 16, 39-46.
- Yamakawa S. (1976). Investigation of peculiarity in some wave-forms through bispectral analysis. *Bulletin of the Japan Society of Mechanical Engineers*, 19, 29-36.
- Yao, N. C. (1974). Bispectral and cross-bispectral analysis of wind and currents off the Oregon Coast. Doctoral dissertation, Oregon State University.
- Zhurbenko, G., and Zuev, N. M. (1975). On higher spectral densities of stationary processes with mixing. *Ukrainian Mathematics Journal*, 27, 364-375.
- Yao, N. C., Neshyba, S., and Crew, H. (1975). Rotary cross-bispectra and energy transfer functions between non-Gaussian vector processes I. Development and example. *Journal of Physical Oceanography*, 5, 164-172.
- Yao, N. C., Neshyba, S., and Crew, H. (1977). Rotary cross-bispectra and energy transfer functions between non-Gaussian vector processes II. Winds and currents off the Oregon coast. *Journal of Physical Oceanography*, 7, 892-903.
- Yasui, S. (1979). Stochastic functional Fourier series, Volterra series, and nonlinear systems analysis. *IEEE Transactions on Automatic Control*, AC-24, 230-242.
- Yeh, T. T., and Van Atta, C. W. (1973). Spectral transfer of scalar and velocity fields in heated-grid turbulence. *Journal of Fluid Mechanics*, 58, Part 2, 233-261.

First authors by year

1953

Blanc-Lapierre, A.

Tukey, J. W.

1954

Magness, T.A.

Mazelsky, B.

1959

Tukey, J. W.

1960

Shiryaev, A. N.

1961

Tick, L. J.

1963

Hasselmann, K.

MacDonald, G.

Sinai, Ya. G.

Tick, L. J.

Tukey, J. W.

1964

Leonov, V. P.

Madden, T.

Rosenblatt, M.

Shaman, P.

1965

Brillinger, D. R.

Godfrey, M. D.

Haubrich, R. A.

Rosenblatt, M.

1966

Akaike, H.

Aubry, M. P.

Hasselmann, K.

Rosenblatt, M.

Van Ness, J. W. (2)

1967

Aubry, M. P.

Bartlett, M. S.

Brillinger, D. R. (2)

Parzen, E.

1968

Bozzi Zadro, M.

Cartwright, D. E.

Hinich, M. J.

Korein, J.

Marussi, A.

1970

Alekseyev, V. G.

Brillinger, D. R.

Dumermuth, G.

Van Atta, C. W.

Westcott, M.

1971

Barnett, T.

Dumermuth, G.

Huber, P. J.

Kleiner, B.

Rosenblatt, M.

1972

Brillinger, D. R.

Dalzell, J. F. (2)

Feuerverger, A.

Gasser, T.

Kiriyama, K.

Mitsuishi, A.

Murty, T. S.

Sasaki, K.

1973

Brillinger, D. R.

Roden, G. I.

Tachi, S.

Yeh, T. T.

1974

Brillinger, D. R. (2)

Mager, P. P.

Van Atta, C. W.

Yao, N. C.

1975

Akaike, H.
Brillinger, D. R.
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Kedem-Kimelfeld, B.
Mager, P. P.
Neshyba, S
Ohta, M.
Sasaki, K
Sato, T.
Tachi, S.
Yao, N. C.
Zhurbenko, G.

1976

Akaike, H.
Briscoe, M. G.
Helland, K. N.
Kim, Y. C.
Lii, K. S.
Lumley, J. L.
Ten Hoopen, M.
Ueno, T.
Yamakawa, S.

1977

Armstrong, J. W.
Brillinger, D. R.
Davies, R. B.
Helland, K. N.
Hung, G.
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Murata, T.
Ohta, M.
Powers, E. J.
Rao, S. T.
Sasaki, K.
Sato, T. (3)
Ten Hoopen, M.
Ueno, T.
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1978

Barresen, R.
Dubkov, A. A.
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Rosenblatt, M.
Sasaki, K.
Sato, T. (3)
Sclove, S. L.
Shimizu, H.
Tanaka, K.

1979

Fried, D. L.
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Helland, K. N.
Hinich, M. J.
Hung, G. (2)
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Kishimoto, T.
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Malakhov, A. N.
Munk, W.
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Nakashima, S. N.
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Ohara, H.
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Rosenblatt, M.
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Sasaki, O.
Sato, T.
Sato, T.
Sato, T.
Sobey, E. J. C.
Stark, L.
Tachi, S.
Takeuchi, K.
Tick, L. J.
Van Atta, C. W.
Van Atta, C. W.
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Barnett, T.
Gabrielli, C.
Tanaka, K.
Powers, E. J.
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Helland, K. N. (2)
Dubkov, A. A.
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Mitsuishi, A.
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Brillinger, D. R. (2)
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Sato, T. (5)
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Sasaki, K. (5)
Sasaki, O.
Neshyba, S.
Hung, G. (2)
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Korein, J.
Helland, K. N.
Yeh, T. T.
Rosenblatt, M.
Ohta, M. (2)
Van Atta, C. W.
Ten Hoopen, M. (2).
Zhurbenko, G.

Brief Discussion or Close Relationship

Tukey (1959) (1963)
Tick (1963)
Bartlett (1967)
Hinich and Clay (1968)
Westcott (1970)
Dumermuth, Huber, Kleiner and Gasser (1970)
Rosenblatt (1971)
Brillinger (1974)
Magar (1974) (1975)
Van Atta (1974)
Ohta, Hatakeyama, Hiromitsu and Yamaguchi (1975)
Kedem-Kimelfeld (1975)
Helland and Van Atta (1976)
Hung and Stark (1977)
Ohta, Yamaguchi, and Hiromitsu (1978)
Sclove (1978)
Yasui (1979)
Gabrielli, Keddum, and Raillon (1979)

General Theory

Blanc-Lapierre and Fortet (1953)
Shiryaev (1960)
Tick (1961)
Hasselmann, Munk and MacDonald (1963)
MacDonald (1963)
Sinai (1963)
Shaman (1964)
Brillinger (1965)
Rosenblatt and Van Ness (1965)
Rosenblatt (1966)
Van Ness (1966)
Akaike (1966)
Parzen (1967)
Brillinger and Rosenblatt (1967a, b)
Huber, Kleiner, Gasser, and Dumermuth (1971)
Sasaki, Sato, and Yamahita (1975)
Brillinger (1975)

Estimation/Computation

Tick (1961)
Hasselmann, Munk and MacDonald (1963)
Shaman (1964)
Brillinger (1965)
Rosenblatt and Van Ness (1965)
Godfrey (1965)
Haubrich (1965)
Rosenblatt (1966)
Van Ness (1966)
Brillinger and Rosenblatt (1967a, b)
Alekseyev (1970)
Van Atta and Yeh (1970)
Huber, Kleiner, Gasser and Dumermuth (1971)
Sasaki, Sato and Yamashita (1975)
Akaike, Arahata, Ozaki (1975)
Dumermuth and Gasser (1978)
Kim and Powers (1979)

Display/Interpretation

Hasselmann, Munk and MacDonald (1963)
MacDonald (1963)
Shaman (1964)
Godfrey (1965)
Haubrich (1965)
Brillinger and Rosenblatt (1967b)
Cartwright (1968)
Marussi, Bozzi Zadro and Manzoni (1968)
Bozzi Zadro and Caputo (1968)
Huber, Kleiner, Gasser and Dumermuth (1971)
Roden and Bendiner (1973)
Neshyba and Sobey (1975)
Lii, Rosenblatt and Van Atta (1976)
Helland, Lii, and Rosenblatt (1977), (1979)
Dumermuth and Gasser (1978)
Kim and Powers (1979)
Kim, Beall, Powers, and Miksad (1980)

Applications

Acoustics

Ohta, Yamaguchi, and Iwashige (1977)
Sasaki, Sato, and Nakamura (1977)
Gerzon (1978)
Sato and Sasaki (1979)

Astrophysics

Aubry (1966), (1967)
Armstrong (1977)
Fried (1979)

Biomedicine

Korein, Tick, Zeitlin, and Randt (1968)
Dumermuth, Huber, Kleiner and Gasser (1970), (1971)
Huber, Kleiner, Gasser, and Dumermuth (1971)
Barnett, Johnson, Naitoh, Hicks and Nute (1971)
Magar (1974)
Tachi (1975)
Ten Hoopen and Zandt (1976), (1977)

Economics

Godfrey (1965)

Fluid Mechanics - Turbulence

Van Atta and Yeh (1970)
Yeh and Van Atta (1973)
Van Atta (1974), (1979)
Helland and Van Atta (1976)
Lumley and Takeuchi (1976)
Lii, Rosenblatt and Van Atta (1976)
Helland, Lii, and Rosenblatt (1977), (1979)
Rosenblatt (1978)
Herring (1980)

Geophysics

Brillinger (1973)
Haubrich (1965)
Hinich and Clay (1968)
Bozzi Zadro and Caputo (1968)
Madden (1964)
Marussi, Bozzi Zadro and Manzoni (1968)

Goodness-of-Fit Tests

Gasser (1975)

Hydromechanics

Hasselmann (1966)
Dalzell (1972a, b)

Image Processing

Sato and Sasaki (1977)
Sato, Sasaki, and Nonaka (1978)
Sasaki, Sato and Nakamura (1978)

Kernel Estimation and Identification

Hung and Stark (1977), (1979)
Hung, Brillinger and Stark (1979)
Yasui (1979)

Mechanical Engineering

Mazelsky (1954)
Yamakawa (1976)
Sato, Sasaki and Nakamura (1977)
Sato and Sasaki (1977)
Ueno and Nakajima (1977)
Sato, Sasaki, and Nonaka (1978)
Shimizu and Inoue (1978)

Meteorology

MacDonald (1963)
Sato, Sasaki, and Mori (1975)

Nonlinear Prediction

Van Ness (1966)

Oceanography

Hasselmann, Munk and MacDonald (1963)
Cartwright (1968)
Murty and Henry (1972)
Roden and Bendiner (1973)
Yao (1974)
Neshyba and Sobey (1975)
Yao, Neshyba and Crew (1975), (1977)
Briscoe (1976)
Rao, Czapski, and Sedefian (1977)
McComas (1978)
Hasselmann (1978)
McComas and Briscoe (1980)

Optics

Sato, Kishimoto and Sasaki (1978)
Sato and Sasaki (1978)
Sasaki, Sato and Oda (1980)

Plasma Physics

Kim and Powers (1976), (1977), (1978), (1979)
Powers and Kim (1977)
Kim, Beall, Powers, and Miksad (1980)

Transfer Function Estimation

Magness (1954)
Tick (1961)
Brillinger (1970)
Tachi (1975)
Kedem-Kimelfeld (1975)
Brillinger (1977)
Borresen (1978)
Hinich (1979)

Signal Processing

Parzen (1967)
Tachi (1973)
Murata and Ohara (1977)
Tanaka, Kikkawa, and Ohara (1978)
Dubkov and Malakhov (1978)

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Sasaki and Sato (1978)

Stochastic Processes

Rosenblatt (1964), (1980)

Brillinger (1972), (1974)

Zhurbenko and Zuev (1975)

Davies (1977)

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The bispectrum or Fourier transform of the 3rd order moments of a time series is useful for the study of nonlinear or non-Gaussian phenomena. This bibliography cites 134 papers covering both theory and application. The entries are classified by content with special effort made to indicate papers that contain material on the computation, display and interpretation of the bispectrum.

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Bispectrum; cumulant spectra; nonlinear time series; polyspectra; spectrum analysis statistics; time series.

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